Taylor's Theorem For one Variable

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If a function $f(x)$ defined in closed interval $[a,b]$ Then by Taylor's Theorem,

$$f(b) = f(a) + (b-a)f'(a) + \frac{(b-a)^2}{2!}f''(a) + \cdots\cdots$$

Maclaurin's series :

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!}f''(0) + \cdots\cdots$$

Q:- Expand the following in powers of x by Maclaurin's series

(i) $\sin x$

(2) $\tan^{-1}x$

Sol: (i) $f(x) = \sin x$ $\qquad f(0) = 0$

$f'(x) = \cos x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x$ $\qquad f''(0) = 0$

$f'''(x) = -\cos x$ $\qquad f'''(0) = -1$

$f^{iv}(x) = \sin x$ $\qquad f^{iv}(0) = 0$

$f^{v}(x) = \cos x$ $\qquad f^{v}(0) = 1$

By Maclaurin's Series,

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5!} f^{v}(0) + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

IInd. part.

let $f(x) = \tan^{-1} x$

$f'(x) = \frac{1}{1+x^2}$

$f''(x) = \frac{-2x}{1+x^2}$

$f'''(x) = -2\frac{(1-3x^2)}{(1+x^2)^3}$

$f^{iv}(x) = \frac{12x(2-2x^2)}{(1+x^2)^4}$

$f^{v}(x) = \frac{24(1-10x^2+5x^4)}{(1+x^2)^5}$

$f(0) = 0$

$f'(0) = 1$

$f''(0) = 0$

$f'''(0) = -2$

$f^{iv}(0) = 0$

$f^{v}(0) = 24 = 4!$

By Maclaurin's Series.

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

$$= 0 + x\cdot 1 + \frac{x^2}{2!}\cdot 0 + \frac{x^3}{3!}(-2!) + \frac{x^4}{4}\cdot 0 + \frac{x^5}{5!}\cdot 4!$$

$$= x - \frac{x^3}{3} + \frac{x^5}{5} \cdots\cdots \infty$$

Ans

26/02/21

Q:- Expand $4x^2 + 5x + 3$ in the power of $(x-1)$ by using Taylor's series.. $\quad a = 1$

Sol.- Here $f(x) = 4x^2 + 5x + 3$

$$f(a) = f(1) = 4(1)^2 + 5(1) + 3 = 12$$

$$f'(x) = 8x + 5 \qquad f'(a) = f'(1) = 8(1) + 5 = 13.$$

$$f''(x) = 8 \qquad f''(a) = f''(1) = 8$$

$$f'''(x) = 0 \qquad f'''(a) = f'''(1) = 0$$

All heigher derivatives of $f(x)$ other than second are vanishes.

Now By Taylor's series expansion

$$f(x) = f(1) + (x-1)f'(1) + \frac{(x-1)^2}{2!}f''(1) + \cdots\cdots$$

$$= 12 + (x-1)(13) + \frac{(x-1)^2}{2!}\cdot 8$$

$$= 12 + 13(x-1) + \overset{2!}{4(x-1)^2}.$$ Ans